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Stochastic Structural Optimization of Multiple Tuned Mass Dampers with Uncertain Bounded System Parameters

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Abstract—*The purpose of the current work is to present the effectiveness of multiple tuned mass dampers (MTMD) to control the vibration of structures under earthquake load. A Stochastic Structural based optimization of Single Tuned Mass Damper (STMD) and Multiple Tuned Mass Dampers (MTMD) parameters in seismic vibration control under bounded uncertain system parameters is presented. The study on MTMD with random parameters in a probabilistic framework is significant. So it is required to accumulate the necessary information about parameters uncertainties. In such cases, the interval method is a feasible alternative. Applying matrix perturbation theory through a first order Taylor series expansion about the mean values of the uncertain parameters' conservative dynamic response bounds are obtained assuming a small degree of parameter uncertainty. The interval extension technique permits the transformation of the problem, initially non-deterministic, into two independents deterministic sub- problems yielding the lower and upper bound solutions. A numerical study is performed to enlighten the effect of parameters' uncertainties on the MTMD parameters' optimization and the safety of the structure. The parametric study is also conducted to define the influence of several parameters (mass ratio, damping ratio of structure) on the effectiveness and robustness of MTMDs considering uniform mass distribution in comparison with single tuned mass damper (STMD).*

1. INTRODUCTION

Lately, due to lack of land space, financial requirements and new developments of construction techniques have induced an increased presence of skyscrapers and other tall structures. The occupants in the upper floors of tall buildings feel discomfort due to structural vibrations caused by dynamic loadings such as earthquake and wind loadings. Thus, mitigating the responses of such structures to external dynamic loads is needed.

Tuned mass damper is the oldest passive vibration control device implemented to mitigate the structural vibration.

The natural frequency of the TMD is tuned in resonance with the fundamental mode of the building structure, so that the huge amount of the structural vibrating energy is transferred to the TMD and dissipated by the damping as the building structure is subjected to earthquake loads. Multiple tuned mass dampers (MTMD) can be proposed in a parallel or series configuration.

The classical approach to designing these structures is to consider deterministic models and parameters, respectively [1,2]. The uncertainties in models and structural parameters, such as mechanical characteristics, external loading and geometric parameters, are introduced in the design process in adopting some simplifying hypotheses. These hypotheses could be either by making use of the parameters' mean values, the use of extreme values, or considering of high level safety factor to ensure the reliability of the designed structures. However, these simplifying assumptions do not yield optimal design in comply with the increasing demand of safer and more economic structures[1,3]. In fact, uncertainties into structure parameters and into external loadings are already there; neglecting their effects on structure response could lead to a non-reliable and/or less economic design; hence, the uncertainties' analysis and response variability must be strongly taken into account to get optimal design. Multiple Tuned Mass Damper (MTMD) [2,4,5] has been studied as passive vibration control problems[6]. It has been demonstrated that MTMD with distributed natural frequencies are more effective than a single TMD. In presence of structural and external loading uncertainties, the main task in designing MTMD parameters is to obtain the appropriate natural frequency and the damping ratio considering these uncertainties. Two categories of methods can be carrying out to describe uncertainties: the probabilistic and the non-probabilistic methods [9]. Nevertheless, these methods cannot be applied when the statistical parameters are insufficient [11,12]. In many practical cases, parameters are only described in a non-probabilistic way by their extreme and mean values and they are called uncertain but bounded (UBB) parameters [6,12,13]. In this context, the convex models and interval analysis methods could be carried out [14]. Interval analysis was firstly used for mechanical engineering in order to solve the extremum of static response for structures in presence of UBB parameters. In such problems, optimization consists in finding the optimal parameters of the TMD in presence of uncertain bounded structural parameters and in presence of stochastic external loading. For the SSO problem, the standard deviation of the structural response will be minimized. Given the fact that optimization problems for SSO involve UBB parameters, the expected optimized solutions should be bounded and defined over an interval. This technique is based on a Taylor expansion followed by an interval extension allows splitting the initial non- deterministic optimization problem (which involves UBB parameters) into two independent deterministic sub-problems involving the upper and the lower bounds solutions. The purpose of this work is to investigate and discuss the efficiency and the validity of this technique for various levels of uncertainties when applied with MTMD optimization strategy: the SSO problems. The technique is based on monotonic assumption of the objective function. The MTMD location for continuous system was also investigated and it has been shown that it has no effects on the optimized results using the studied method.

2. THEORETICAL FORMULATION

2.1. Designing of MTMD

The aim of designing MTMD is to tune damper parameters to the fundamental mode of vibration. It means that the natural damper frequency (or a group of dampers) must be close to the natural frequency of fundamental vibration mode of structure. Moreover, the damping coefficient of the damper must be appropriately chosen by (Zuo and Nayfeh, 2005) and is obtained using equations developed by Den Hartog (1956) for the SDOF damper.The optimum parameters of such a damper (or group of MTMD) can be obtained from the formulae given in a paper (Warburton 1982). The optimal frequency ratio is determined from:

$$\frac{\omega_d^2}{\omega_s^2} = \frac{2+\mu}{2(1+\mu)^2} \tag{1}$$

Where,

$$\mu = \frac{\sum_{j=1}^{n} m_j}{M_s}, \omega_s^2 = \frac{K_s}{M_s}, \omega_d^2 = \frac{k_d}{m_d}, m_d = \sum_{j=1}^{n} m_j \tag{2}$$

2.2. The Dynamic Equation of Motion of Structure and MTMD System

The equation of motion [2,9] of a SDOF system attached with MTMD (as shown in Fig.1) can be expressed as,

$$M\ddot{X} + C\dot{X} + KX = -M\bar{r}\ddot{z}_b \tag{3}$$

Where, $X = [x_s, x_1, x_2, \dots x_n]^T$ is the relative displacement vector, and $\bar{r} = [0\ I]^T$, where I is an nx1 unit vector. M, C and K represent the mass, damping and stiffness matrix of the combined system.

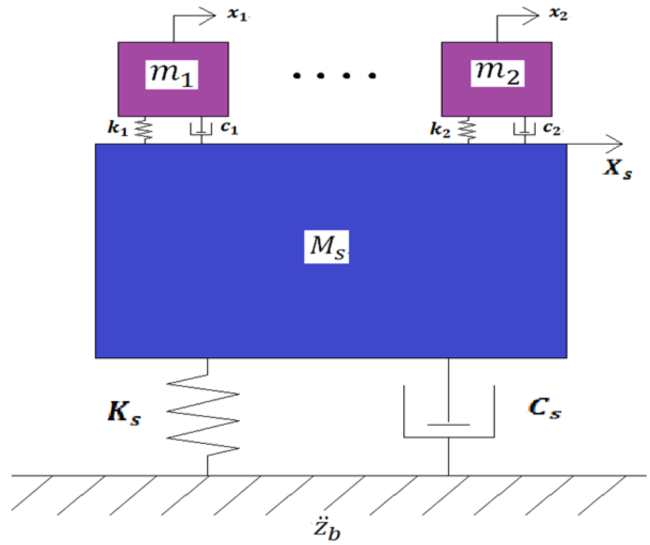

Fig. 1: Structure-MTMD System

$$M = \begin{bmatrix} M_s & 0 \\ 0 & m \end{bmatrix} \tag{4}$$

Where, M_s is the mass of structure and m is the matrix of dampers.

$$m = diag[m_1 m_2 \dots m_n]$$

$$K = \begin{bmatrix} K_s + k_d & k^{\#} \\ k^{\#T} & k \end{bmatrix} \tag{5}$$

Where, K_s is the stiffness of structure.

$$K_d = \sum_{j=1}^{n} k_j$$

$$k^{\#} = [-k_1 \quad -k_2 \quad \dots -k_n]$$

$$k = diag[k_1 \quad k_2 \quad \dots k_n]$$

The damping matrix of the system C is in a form similar to that of the stiffness matrix K.

$$C = \begin{bmatrix} C_s + c_d & c^{\#} \\ c^{\#T} & c \end{bmatrix} \tag{6}$$

Where, C_s is the damping of structure.

$$c^{\#} = [-c_1 \quad -c_2 \quad \dots -c_n]$$

$$\frac{c_j}{2\sqrt{m_j k_j}} = \sqrt{\frac{3\mu_j}{8(1+\mu_j)}}, \varepsilon_j = \frac{c_j}{2m_j\omega_j}$$

$$c = diag[c_1 \quad c_2 \quad \dots c_n]$$

Introducing the state space vector, $X_s = [x_d, x_s, x_f, \dot{x}_d, \dot{x}_s, \dot{x}_f]^T$

$$\dot{X}_s = A_s X_s + \bar{r}\ddot{z}_b \tag{7}$$

Where, $A_s = \begin{bmatrix} 0 & I \\ H_k & H_c \end{bmatrix}$

$$H_k = -M^{-1}K \text{ and } H_c = -M^{-1}C$$

In which $\bar{r} = [0 \quad I]^T$ with **I** and 0 is the (n+1)x(n+1) unit and null matrices, respectively.

2.3. Determination of Response Covariance

The structure-MTMD system as shown in **Fig.1** is subjected to stochastic load due to the random seismic acceleration that excites the primary structure at base. A widely adopted stationary model of $\ddot{z}_b(t)$ is obtained by filtering a white noise process acting at the bed rock through a linear filter which represents the surface ground. This is the well-known Kanai–Tajimi stochastic process [Tajimi 1960] [6] which is able to characterize the input frequency content for a wide range of practical situations. The process of excitation at the base can be described as: $\ddot{x}_f(t) + 2\varepsilon_f\omega_f\dot{x}_f + \omega_f^2 x_f = -\omega(t)$ and

$$\ddot{y}_b(t) = \ddot{x}_f(t) + \omega(t) = 2\varepsilon_f\omega_f\dot{x}_f + \omega_f^2 x_f \quad \textbf{(8)}$$

Where, $\omega(t)$ is a stationary Gaussian zero mean white noise process, representing the excitation at the bed rock, ω_f is the base filter frequency and ε_f is the filter or around damping. Defining the global state space vector is defined as:

$$Z = [y, x, x_f, \dot{y}, \dot{x}, \dot{x}_f]^T$$

Eqn. (7) and (8) leads to an algebraic matrix equation of order six i.e. the so called Lyapunov equation (Lutes and Sarkani 2001):

$$AR + RA^T + B = 0 \quad (9)$$

The details of the state space matrix A and B in Eqn. (9) are as below:

$$[A] = \begin{bmatrix} 0 & I \\ \hat{H}_k & \hat{H}_c \end{bmatrix} \quad (10)$$

Where, $\hat{H}_k = \begin{bmatrix} & \vdots\ \omega_f^2 \\ -M^{-1}K & \vdots\ \omega_f^2 \\ & \vdots \\ & \vdots \\ & \omega_f^2 \\ \dots\ \dots\ \dots & \dots\ \dots\ \dots \\ 0\ 0\ \dots\ 0 & \vdots\ -\omega_f^2 \end{bmatrix}$

$$\hat{H}_c = \begin{bmatrix} & \vdots\ 2\varepsilon_f\omega_f \\ -M^{-1}C & \vdots\ 2\varepsilon_f\omega_f \\ & \vdots \\ & \vdots \\ & 2\varepsilon_f\omega_f \\ \dots\ \dots\ \dots & \dots\ \dots\ \dots \\ 0\ 0\ \dots\ 0 & \vdots\ -2\varepsilon_f\omega_f \end{bmatrix}$$

$$[B] = \begin{bmatrix} 0 & \dots & \dots & \dots & \dots & 0 \\ \vdots & \dots & \dots & \dots & \dots & \vdots \\ \vdots & \dots & \dots & \dots & \dots & \vdots \\ \vdots & \dots & \dots & \dots & \dots & \vdots \\ \vdots & \dots & \dots & \dots & \dots & \vdots \\ 0 & \dots & \dots & \dots & \dots & 2\pi S_o \end{bmatrix} \quad (11)$$

The space state covariance matrix R is obtained as the solution of the Lyapunov equation. The state space covariance matrix R is obtained as solution of the Lyapunov equation. It is represented as:

$$R = \begin{bmatrix} R_{zz} & R_{z\dot{z}} \\ R_{\dot{z}z} & R_{\dot{z}\dot{z}} \end{bmatrix} \quad (12)$$

The root mean square of displacement (RMSD) of the TMD and that of the primary structure can be obtained as:

$$\sigma_{x_o} = \sqrt{R_{zz}(2,2)} \quad (13)$$

In the following sections, besides the stochastic loading, the system matrices (A, B) defined in Eq. (9) will be considered as uncertain and the TMD parameters will be optimized in order to minimize some objective function.

When structure parameters $x = (x_1, x_2, x_3, \dots x_3)$, q is the number of uncertain parameters, are UBB parameters, it is convenient to describe them using intervals. Let $X = [X_1, X_2, X_3, \dots X_q]$ be the corresponding box then for every $x_i \in x$ the corresponding interval is $X_i = [\underline{x}_i, \overline{x}_i]$. Introducing the mean value μ_i of X_i and the maximum deviation Δx_i from the mean, the uncertain but bounded parameter can be written as:

$$X_i = [\underline{x}_i, \overline{x}_i] = [\mu_i - \Delta x_i, \mu_i + \Delta x_i] = \mu_i + e_\Delta \Delta x_i$$

where $e_\Delta = [-1, 1]$

Then, the i[th] such interval variable [6] can be defined as

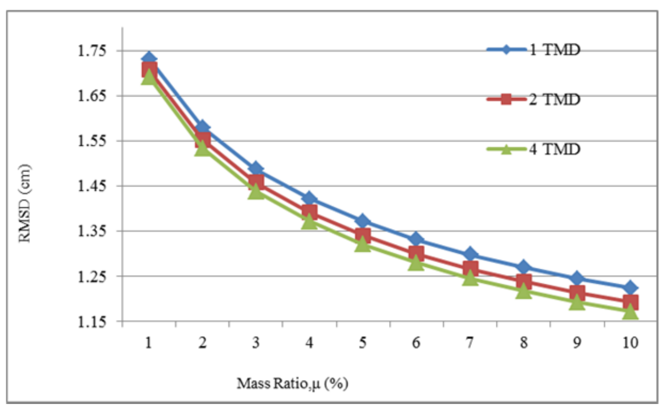

Fig. 2: The variation of RMSD with a varying mass ratio

$$x_i = \mu_i + \delta x_i \ where\ |\delta x_i| \le \Delta x_i, i = 1 \dots\dots q$$

Any response variable **f(x)** that depends on UBB parameter **X** is also UBB response. Assuming that the level of uncertainties is small, the response can be expanded in the Taylor series about the mean value $\boldsymbol{\mu} = (\boldsymbol{\mu_1} \ldots\ldots \boldsymbol{\mu_q})$ in the first order terms of $\boldsymbol{\delta x_i} \in [-\Delta x_i, \Delta x_i]$ as:

$$\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{\mu}) + \sum_{i=1}^{q} \frac{\partial f}{\partial x_i} \boldsymbol{\delta x_i} + \ldots \tag{14}$$

By making use of the interval extension [6,11,12] in interval mathematics and adopting the monotonic assumption, the interval region of the function involving the UBB parameters can be separated out to the upper and lower bound function as follows:

$$\overline{\boldsymbol{f}}(\boldsymbol{X}) = \boldsymbol{f}(\boldsymbol{\mu}) + \sum_{i=1}^{q} \frac{\partial f}{\partial x_i} \boldsymbol{\delta x_i} + \ldots \text{ and}$$

$$\underline{\boldsymbol{f}}(\boldsymbol{X}) = \boldsymbol{f}(\boldsymbol{\mu}) - \sum_{i=1}^{q} \frac{\partial f}{\partial x_i} \boldsymbol{\delta x_i} + \ldots \tag{15}$$

In the above, the interval region of the objective function involving the UBB parameters is separated out to upper bound and the lower bound function. Thus, the optimization problem now involves two separate objective functions correspond to the lower bound and the upper bound solutions.

3. OPTIMIZATION STRATEGY, THE SSO

The system matrices **A** and **B**, defined in Eq. **(9)**, involve UBB parameters then the associated response covariance matrix **R** will also involve these parameters. The system matrices can be approximated using Taylor expansion and written in the form of Eq. **(15)**. From Eq. **(9)**, one can obtain the after neglecting the higher order term in the first order Taylor series following equations:

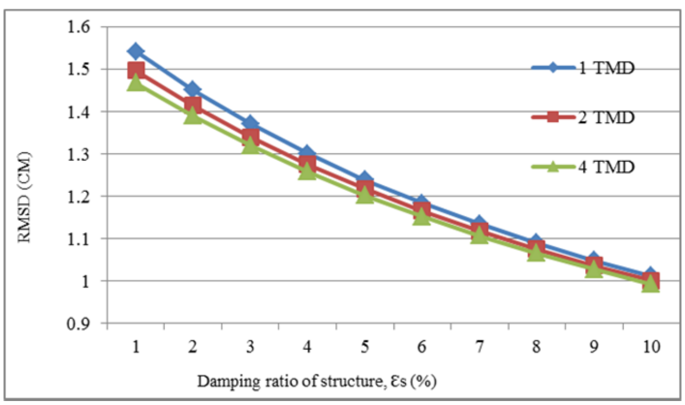

Fig. 3: The variation of RMSD with a varying damping ratio of structure

$$\boldsymbol{A_o R_o} + \boldsymbol{R_o A_o^T} + \boldsymbol{B_o} = \boldsymbol{0} \tag{16}$$

$$\boldsymbol{A_o} \frac{\partial \boldsymbol{R}}{\partial x_i} + \frac{\partial \boldsymbol{R}}{\partial x_i} \boldsymbol{A_o^T} + \boldsymbol{B'} = \boldsymbol{0} \tag{17}$$

$$\boldsymbol{B'} = \frac{\partial \boldsymbol{A}}{\partial x_i} \boldsymbol{R_o} + \boldsymbol{R_o} \frac{\partial \boldsymbol{A^T}}{\partial x_i} + \frac{\partial \boldsymbol{B}}{\partial x_i} \tag{18}$$

It can be further noted that the RMSD of the primary structure as defined by Eq. **(13)** is also a function of UBB parameters and can be expanded in the first order Taylor series as the mean and fluctuating part as below:

$$\boldsymbol{\sigma_x} = \boldsymbol{\sigma_{x_o}} + \sum_{i=1}^{m} \frac{\partial \sigma_x}{\partial x_i} \boldsymbol{\delta_{x_i}} + \ldots \tag{19}$$

The sensitivity of the RMSD of the primary structure can be readily obtained by differentiating Eq. **(13)** w.r.t i[th] uncertain parameter as:

$$\frac{\partial \sigma_x}{\partial x_i} = \frac{1}{2} \frac{\frac{\partial R(2,2)}{\partial x_i}}{\sqrt{R(2,2)}} \tag{20}$$

In which, $\frac{\partial \sigma_x}{\partial x_i}$ can be obtained by solving Eq. **(19)**. The interval stiffness and mass matrix can be obtained directly with the interval parameters. By using the interval extension in the interval mathematics, the interval extention of Eq. **(20)** can be obtained as :

$$\boldsymbol{\sigma_x^u} = \boldsymbol{\sigma_o} + \sum_{i=1}^{m} \frac{\partial \sigma_x}{\partial x_i} \Delta \boldsymbol{x_i} + \ldots\text{and}$$

$$\boldsymbol{\sigma_x^l} = \boldsymbol{\sigma_o} - \sum_{i=1}^{m} \frac{\partial \sigma_x}{\partial x_i} \Delta \boldsymbol{x_i} + \ldots \tag{21}$$

In the above, the interval region of the objective function involving the UBB parameters is separated out to upper bound and the lower bound function. Thus, the optimization problem now involves two separate objective functions correspond to the lower bound and the upper bound solutions.

4. NUMERICAL STUDY

A TMD attached to a primary system is undertaken to elucidate the proposed TMD parameter's optimization procedure considering UBB type system parameters. The uncertainties are considered in $\omega_f, \varepsilon_f, \omega_s, \varepsilon_s$ *and* S_o. The uncertainties in any such i-th parameter, X_i are described by ΔX_i representing the maximum possible dispersion expressed in terms of the percentage of corresponding nominal value ($\bar{X}_i$). Unless mentioned otherwise, the following nominal values are assumed for the present numerical study:

$\omega_f = 9\pi\ rad/s$, $\varepsilon_f = 0.4$, $\omega_s = 6\pi\ rad/s$, $\varepsilon_s = 0.03$ $S_o = 0.03\ m^2 s^{-3}$, $\mu = 0.05$.

The optimized results for STMD & MTMD will be compared with respect to the deterministic optimization, where mean values of uncertain parameters are taken. In the present study, optimizations are performed using the genetic algorithm available with the MATLAB Global Optimization Toolbox.

Using proposed optimization procedure considering the upper and lower bound objective function as represented by Eq. **(21)**, the optimum TMD parameters are obtained.

The mass ratio taken here is 5%. Fig. 2 and Fig. 3 show the variation of RMSD with a varying mass ratio and damping ratio of structure respectively for 1, 2 and 4 TMDs. It clearly proves that increasing the value of Mass ratio or Damping ratio of structure will lead to the decrease in the value of RMSD. In addition to this, as number of TMDs (MTMD) increases, the value of RMSD decreases which clearly proves that MTMD is more useful in mitigating the vibrations in a structure than STMD.

Fig. 4, Fig. 5 and Fig. 6 shows the graph of different parameters with their deterministic value and upper bound and lower bound value for single tuned mass damper (STMD).

Fig. 7, Fig. 8 and Fig. 9 shows the graph of different parameters with their deterministic value and upper bound and lower bound value for two tuned mass damper (MTMD). Thus examining the graphs of both STMD & MTMD, it is clear that there is a decrease in the RMSD value when single TMD is replaced by two TMDs. As expected, the optimum TMD parameter and the probability of failure results considering a deterministic system parameter is within the bounded solution. However the width of the bounded solution band increases as the level of uncertainty increases.

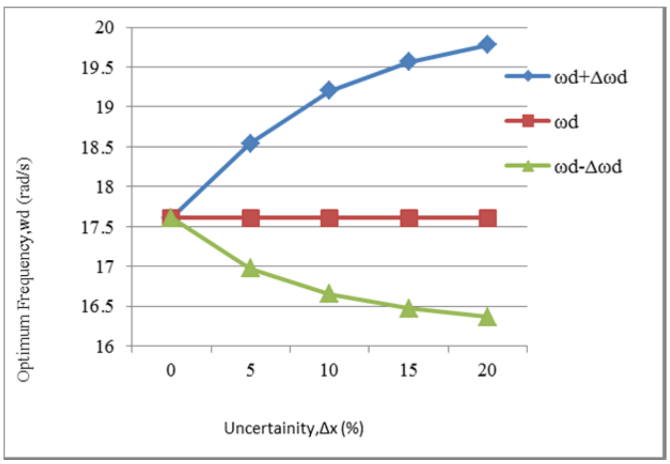

Fig. 4: The variation of Optimum Frequency, ω_d with a varying level of uncertainty, ΔX (1 TMD)

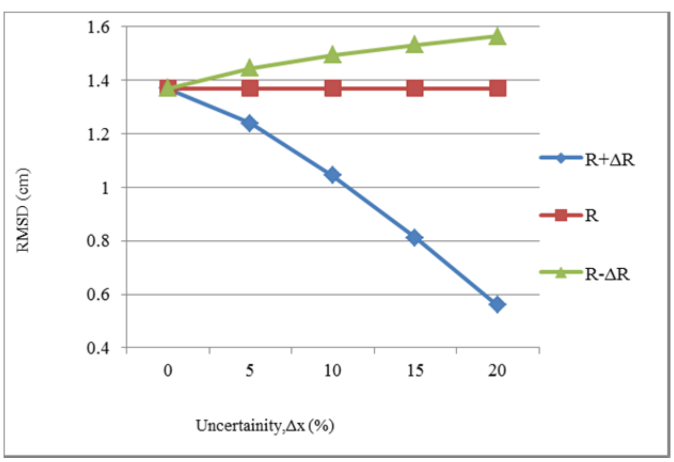

Fig. 5: The variation of RMSD with a varying level of uncertainty, ΔX (1 TMD)

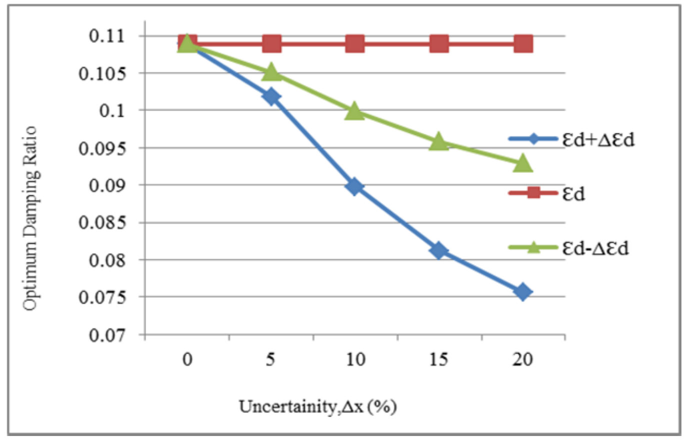

Fig.6 The variation of Optimum Damping Ratio, ε_d with a varying level of uncertainty, ΔX (1 TMD)

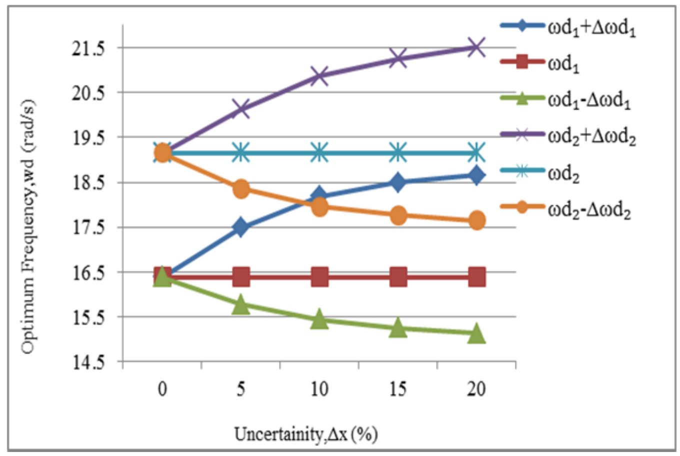

Fig. 7: The variation of Optimum Frequency, ω_d with a varying level of uncertainty, ΔX (2 TMD)

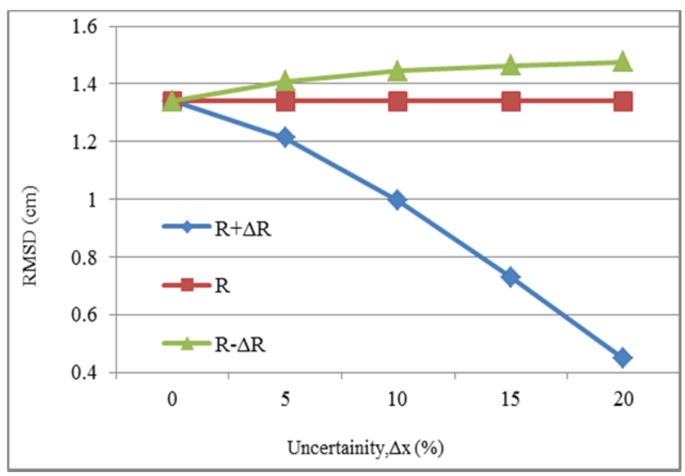

Fig. 8: The variation of RMSD with a varying level of uncertainty, ΔX (2 TMD)

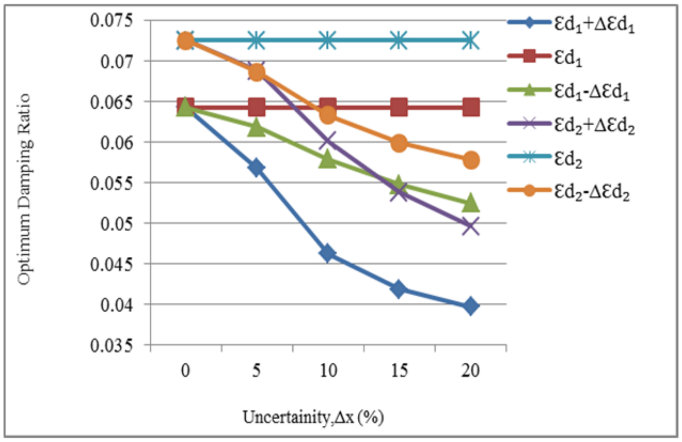

Fig. 9: The variation of Optimum Damping Ratio, ε_d with a varying level of uncertainty, ΔX (2 TMD)

5. CONCLUSION

In this paper, a MTMD model is studied and the results are compared with the STMD. Optimization strategies used is the stochastic structural optimization (SSO). In presence of uncertain bounded structural parameters, the method permits splitting the initial non deterministic problem into two deterministic independent sub-problems. The optimized upper and lower bounds of the root mean square displacement in the SSO problems are linear functions of uncertainties and they are divergent of the deterministic optimized value, which is completely reasonable because larger is the range of uncertainties, larger is the range of optimized bounds. These results still valid even for different locations of the TMD on the beam, only the deterministic value is affected and both bounds are in one side and in the other of the deterministic value. According to the obtained results for a STMD & MTMD, it is clear that MTMD helps in mitigating the vibrations in a structure than STMD.

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